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Inverse Split and Non split Domination in Fuzzy graphs

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ABSTRACT
In this paper we define the notions of inverse split and non split domination in fuzzy graphs. We get many bounds on inverse split and non split domination numbers.
Keywords – dominating set, split dominating set, inverse split dominating set, non split dominating set, inverse non split dominating set.

I. INTRODUCTION

Rosenfeld introduced the concept of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness.Nagoorgani and Chandrasekaran discussed domination in fuzzy graph using strong arcs.Kulli V.R introduced the concept of split domination and non split domination in graphs. This paper deals with inverse split and non-split domination in fuzzy graphs

Definition: 1.1

A non empty set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is a dominating set of G if every vertex in V-D is adjacent to some vertex in D.The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G.

Definition: 1.2

Let D be the minimum dominating set of G.If V-D contains a dominating set $D^{'}$ then $D^{'}$ is called the inverse dominating set of G with respect to D.The inverse domination number $\gamma^{'}(G)$ is the minimum cardinality taken over all the minimal inverse dominating set of G.

Definition: 13

A dominating set of a fuzzy graph G is a split (non split) dominating set if the induced subgraph $\langle V - D \rangle$ is disconnected (connected).

Definition: 1.4

The split (non split) domination number $\gamma_s(G)[\gamma_{ns}(G)]$ is the minimum cardinality of a split(non split) dominating set.

Definition: 1.5

Two nodes that are joined by a path are said to be connected.

II MAIN RESULTS

Definition: 2.1

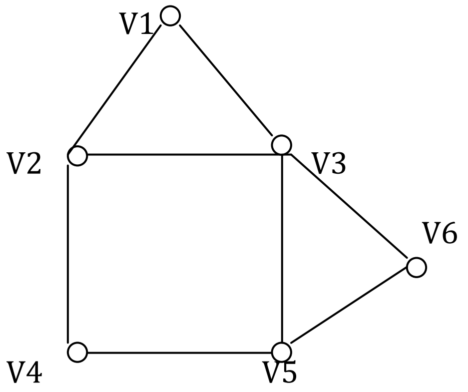
Let $D^{'}$ be the minimum inverse dominating set of G with respect to D.Then $D^{'}$ is called an inverse split(non split)dominating set of G if the induced subgraph $\langle V - D^{'} \rangle is$ disconnected(connected). The inverse split (non-split) domination number is denoted by $\gamma_s^{'}(G)(\gamma_{ns}^{'}(G))$ and is the minimum cardinality taken over all the minimal inverse split (non-split)dominating sets of G.Bounds on $\gamma_s^{'}(G) and \gamma_{ns}^{'}(G)$ are also obtained.

Remark:

i)For any complete fuzzy graph K_n with $n \geq 2$ vertices $\gamma_s^{'}(K_n) = 0 and \gamma_{ns}^{'}(K_n) \leq 1$

ii) $\gamma_{ns}^{'}(C_n) = 0$

EXAMPLE: 2.2



$\sigma(v1)=0.5;\sigma(v2)=0.9;\sigma(v3)=0.4;\sigma(v4)=0.8;$
$\sigma(v5)=0.6;\sigma(v6)=0.3$

$D=\{V2,V5\}$

$D^{'}=\{V3,V4\}$

$\left\langle V-D^{'}\right\rangle$ *isdisconnected*

THEOREM: 2.3

For any fuzzy graph G,

$$\gamma^{'}(G)\le\gamma_{s}^{'}(G)$$
$$\gamma^{'}(G)\le\gamma_{ns}^{'}(G)$$

Proof:

Since every split dominating set of G is an inverse dominating set of G,we have $\gamma^{'}(G)\le\gamma_{s}^{'}(G)$.Similarly,every inverse non-split dominating set of G is an inverse dominating

set of G,we have $\gamma^{'}(G)\le\gamma_{ns}^{'}(G)$.

THEOREM: 2.4

For any fuzzy graph G $\gamma^{'}(G)\le\min\{\gamma_{s}^{'}(G),\gamma_{ns}^{'}(G)\}$

Proof:

Since every inverse split dominating set

and every inverse non-split dominating set of G are the inverse dominating sets of G,we have $\gamma^{'}(G)\le\gamma_{s}^{'}(G)$ *and* $\gamma^{'}(G)\le\gamma_{ns}^{'}(G)$

THEOREM: 2.5

Let T be a fuzzy tree such that any two adjacent cut vertices u and v with at least one of u and v is adjacent to an end vertex then $\gamma^{'}(T)=\gamma_{s}^{'}(T)$

Proof :

Let $D^{'}$ be a $\gamma^{'}$ set of T then we consider the following two cases:

case i) Suppose that atleast one of $u,v\in D^{'}$, then $\left\langle V-D^{'}\right\rangle$ is disconnected with atleast one vertex.Hence $D^{'}$ is a $\gamma_{s}^{'}$ set of T.Thus the theorem is true.

caseii) Suppose $u,v\in V-D^{'}$.Since there exists an end vertex w adjacent to either u or v say u,it implies that $w\in D^{'}$.Thus it follows that $D^{''}=D^{'}-\{w\}\cup\{u\}$ is a $\gamma^{'}$ set of T.Hence by case i) the theorem is true.

THEOREM: 2.6

For any fuzzy tree T, $\gamma_{ns}^{'}(T)=n-p$ where p is the number of vertices adjacent to end vertices.

THEOREM: 2.7

For any fuzzy graph G, $\gamma_{ns}^{'}(G)\le n-\delta(G)$, where $\delta(G)$ is the minimum degree among the vertices of G

Remark:

1.For any fuzzy tree T, $\delta(T)\le 1$

2. If H is any connected spanning subgraph of G,then $\gamma^{'}(G)\le\gamma^{'}(H)$

THEOREM: 2.8

Let G be a fuzzy graph which is not a cycle wirth atleast 5 vertices.Let H be a spanning subgraph of G then $\gamma_{s}^{'}(G)\le\gamma_{s}^{'}(H)$ *and* $\gamma_{ns}^{'}(G)\le\gamma_{ns}^{'}(H)$

Proof:

Since G is connected then any spanning tree T of G is minimally connected subgraph of G such that
$\gamma_s^{'}(G) \le \gamma_s^{'}(T) \le \gamma_s^{'}(H)$

Similarly $\gamma_{ns}^{'}(G) \le \gamma_{ns}^{'}(T) \le \gamma_{ns}^{'}(H)$

THEOREM:2.9

If T is a fuzzy tree which is not a star then
$\gamma_{ns}^{'}(T) \le n-2 \; for\, all\, n \ge 3$

Proof:

Since T is not a star,There exists two adjacent cut vertices u and v with degree u and degree $v \ge 2$. This implies that $V-\{u,v\}$ is an inverse non split dominating set of T.Thus the theorem is true

THEOREM:2.10

An inverse non-split dominating set $D^{'}$ of G is minimal iff for each vertex $v \in D^{'}$ one of the following conditions is satisfied

i) There exists a vertex u $u \in V - D^{'} such\, that$ N(u)$\cap D^{'} = \{v\}$

ii) v is not an isolated vertex in $\langle D^{'} \rangle$

iii) u is not an isolated vertex in $\langle V - D^{'} \rangle$

Proof: Suppose that $D^{'}$ is a minimal inverse non-split dominating set of G. Suppose the contrary. That is v does not satisfy any of the given conditions. Then there exists an inverse dominating set $D^{''} = D^{'} - \{v\}$ such that the induced subgraph $\langle V - D^{''} \rangle$ is connected. This implies that $D^{''}$ is an inverse non-split dominating set of G contradicting the minimality of $D^{'}$.

Therefore the condition is necessary.

III COEDGE SPLIT AND NON-SPLIT DOMINATING SETS FOR FUZZY GRAPH G

Definition:3.1

A subset X of E is an edge dominating set (ED-set) if every edge in E\X is adjacent to some edge in X.The minimum cardinality of the minimal ED-set of G is called the edge domination number of G and is denoted by $\gamma^{'}(G)$

Definition:3.2

Let X be the minimum edge dominating set of G.If E\X contains an edge dominating set X_1, then X_1 is called the complementary edge dominating set (or)co-edge dominating set of G with respect to X.The co-edge domination number is the minimum cardinality of the minimal co-edge dominating set of G.

Definition:3.3

An edge dominating set X of a fuzzy graph G is called a connected edge dominating set (CED) if the edge induced subgraph $\langle X \rangle$ is connected.The minimum cardinality of a minimal CED set of G is called the connected edge domination number and it is denoted by $\gamma_c^{'}(G)$.

Definition:3.4

A coedge split dominating set (CESD) of a fuzzy graph G is a coedge dominating set X of a fuzzy graph G such that the induces subgraph $\langle E \setminus X \rangle$ is disconnected and the coedge split domination number $\gamma_{cs}^{'}(G)$ is the minimum cardinality of the minimal coedge split dominating set of G.

Definition:3.5

A coedge dominating set X of a fuzzy graph G is a coedge non split dominating set (CENSD) if the edge induced subgraph $\langle E \setminus X \rangle$ is connected. The coedge nonsplit domination number is denoted by $\gamma_{cens}^{'}(G)$ and it is the minimum cardinality of the minimal coedge nonsplit dominating set of G.

THEOREM:3.6

For any fuzzy graph G, $\gamma^{'}(G) \le \gamma_{cens}^{'}(G)$

Proof:

Since every CENSD set of G is an ED set of G.Hence the result.

THEOREM:3.7

A CENSD set of G is minimal iff for each edge $e \in X$, one of the following conditions is satisfied.

i) There exists an edge $f \in E \setminus X \text{ such that } N(f) \cap X = \{e\}$

ii) e is an isolated edge in $\langle X \rangle$ and

iii) $N(e) \cap (E \setminus X) = \varphi$

Proof:Let X be a CENSD set of G.Assume that X is minimal. Therefore , $X - \{g\}$ is not a CENSD set for any $g \in X$ .Now to provew that any one of the above three conditions is satisfied. On the contrary , if there exists an edge $e \in X$ such that e does not satisfy any of the given conditions then $X' = X - \{e\}$ is an ED set of G. Also $N(e) \cap (E \setminus X) \neq \varphi$ gives $\langle E \setminus X' \rangle$ is connected. This implies that X' is a CENSD set of G which contradicts the minimality of X. This proves the necessity.Conversely,for connected fuzzy graph ,if any one of the given three conditions is satisfied gives the sufficiency.

THEOREM:3.8

If X is a minimal ED set of a fuzzy graph G then E\X is an ED set of G.

Proof:

Let f be any edge of X.If f is not adjacent to some edge g in X and since G has no isolated edges, f is adjacent to some edge g in X.In this case $X - \{f\}$ is an ED set which contradicts the ninimality of X.Hence E\X is an ED set of G.

THEOREM:3.9

Every edge of a fuzzy graph is a minimal ED set iff the graph is a star graph

Proof:

Since every edge of a graph is minimal ED set of G must be adjacent to each other edge .Hence G must be a star graph. The converse follows immediately.

Conclusion

The concept of domination in fuzzy graphs rich in theoretical developments and applications.Many domination parameters have been investigated by different authors.Whenever, D is a dominating, V-D is also a dominating set.In an information retrieval system, we always have a set of secondary nodes ,to do the job in the complement.Thus the dominating sets and the inverse dominating sets can stand together to facilitate the communication process.They play very important role in coding theory,computer science,operations research, switching circuits, electrical circuits.

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